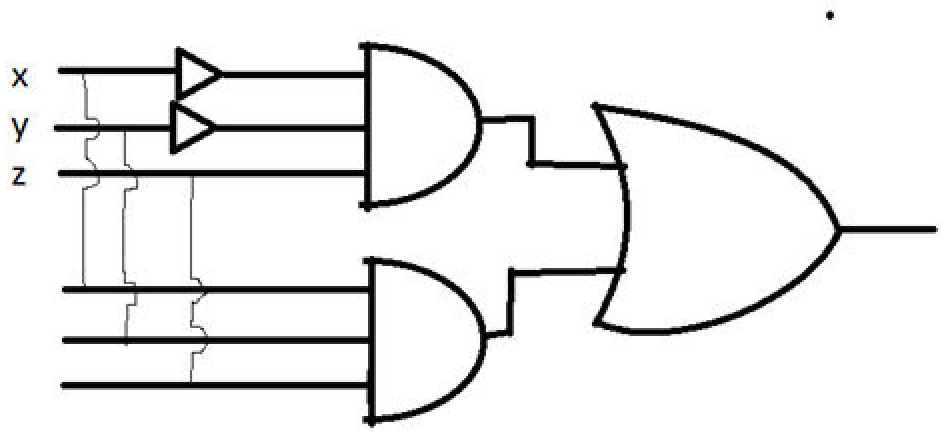
MCS 013

July 2016-january 2017 session

Ans 3 (a)

(a) Make logic circuit for the following Boolean expressions:

(i) (x' y' z) + (xyz)



(b) Find Boolean Expression of Q in the figure given below.

Figure 1: Boolean Circuit.

sol:

(A.B) + (B+C).(C.B)

(c) Find Boolean Expression of Q in the figure given below.

Figure 2: Boolean Circuit

sol.

(X.Y)' + Z' + Y

(d) What is integer partition? Write down all partitions of 8.

sol .

Ans

In number theory and combinatorics, a **partition** of a positive integer *n*, also called an **integer partition**, is a way of writing *n* as a sum of positive integers. Two sums that differ only in the order of theirsummands are considered the same partition. (If order matters, the sum becomes a composition.) For example, 4 can be partitioned in five distinct ways:

4
3 + 1
2 + 2
2 + 1 + 1
1 + 1 + 1 + 1

The order-dependent composition 1 + 3 is the same partition as 3 + 1, while the two distinct compositions 1 + 2 + 1 and 1 + 1 + 2 represent the same partition 2 + 1 + 1.

A summand in a partition is also called a **part**. The number of partitions of *n* is given by the partition function $p(n)$. So $p(4) = 5$. The notation $\lambda \vdash n$ means that λ is a partition of *n*.

Partitions can be graphically visualized with Young diagrams or Ferrers diagrams. They occur in a number of branches of mathematics andphysics, including the study of symmetric polynomials, the symmetric group and in group representation theory in general.

Write down all partitions of 8

8

7+1

6+2

5+2+1

4+2+1+1

3+2+1+1+1

2+2+1+1+1+1

1+1+1+1+1+1+1+1

4. (a) How many different committees can be formed of 10 professionals, each containing at least 4 Professors, at least 3 General Managers and 3 Finance Advisors from list of 10 Professors, 12 General Managers and 5 Finance Advisors?

PROFESSOR	GENERAL MANAGER	FINANCE ADVISORS
4	3	3

SO
SOLUTION IS
C(10,4) + C(12,3) + C(5,3)
(10X9X8X7)/(4X3X2X1)+(12X11X10)/(3X2X1)+(5X2)/(2X1)
(10X3X7) +(11X10X2) + 5
210+220+5
435

(b) There are two mutually exclusive events A and B with P(A) =0.5 and P(B) = 0.4. Find the probability of followings:

(i) A does not occur

P(A')=1-P(A)=1-0.5=0.5

(ii) Both A and B does not occur

P(A')=1-P(A)=1-0.5=0.5

P(B')=1-P(B)=1-0.4=0.6

P(A' AND B') = P(A') X P(B') =0.5 X 0.6 = 0.3

(iii) Either A or B does not occur

P(A')=1-P(A)=1-0.5=0.5

P(B')=1-P(B)=1-0.4=0.6

P(A' OR B') = P(A') + P(B') - P(A' AND B') =0.5 + 0.6 - 0.3 = 0.9

(c) What is set? Explain the basic properties of sets.

Ans

A set is a well defined collection of distinct objects. The objects that make up a set (also known as the elements or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. Georg Cantor, the founder of set theory, gave the following definition of a set at the beginning of his Beiträge zur Begründung der transfiniten Mengenlehre:[1]

A set is a gathering together into a whole of definite, distinct objects of our perception [Anschauung] or of our thought—which are calledelements of the set.

Sets are conventionally denoted with capital letters. Sets A and B are equal if and only if they have precisely the same elements.[2]

Cantor's definition turned out to be inadequate for formal mathematics; instead, the notion of a "set" is taken as a primitive notion in axiomatic set theory, and the properties of sets are defined by a collection of axioms. The most basic properties are that a set can have elements, and that two sets are equal (one and the same) if and only if every element of each set is an element of the other; this property is called the extensionality of sets.

property

The two basic properties to represent a set are explained below using various examples.

1. The change in order of writing the elements does not make any changes in the set.

In other words the order in which the elements of a set are written is not important. Thus, the set {a, b, c} can also be written as {a, c, b} or {b, c, a} or {b, a, c} or {c, a, b} or {c, b, a}.

For Example:

Set A = {4, 6, 7, 8, 9} is same as set A = {8, 4, 9, 7, 6}

i.e., {4, 6, 7, 8, 9} = {8, 4, 9, 7, 6}

Similarly, {w, x, y, z} = {x, z, w, y} = {z, w, x, y} and so on.

2. If one or many elements of a set are repeated, the set remains the same.

In other words the elements of a set should be distinct. So, if any element of a set is repeated number of times in the set, we consider it as a single element. Thus, {1, 1, 2, 2, 3, 3, 4, 4, 4} = {1, 2, 3, 4}

The set of letters in the word 'GOOGLE' = {G, O, L, E}

For Example:

The set A = {5, 6, 7, 6, 8, 5, 9} is same as set A= {5, 6, 7, 8, 9}

i.e., {5, 6, 7, 6, 8, 5, 9} = {5, 6, 7, 8, 9}

In general, the elements of a set are not repeated. Thus,

(i) if T is a set of letters of the word 'moon': then T = {m, o, n},

There are two o's in the word 'moon' but it is written in the set only once.

(ii) if U = {letters of the word 'COMMITTEE'}; then U = {C, O, M, T, E}

Solved examples using the properties of sets:

1. Write the set of vowels used in the word 'UNIVERSITY'.

Solution:

Set V = {U, I, E}

2. For each statement, given below, state whether it is true or false along with the explanations.

(i) {9, 9, 9, 9, 9, ……..} = {9}

(ii) {p, q, r, s, t} = {t, s, r, q, p}

Solution:

(i) {9, 9, 9, 9, 9, ……..} = {9}

True, since repetition of elements does not change the set.

(ii) {p, q, r, s, t} = {t, s, r, q, p}

True, since the change in order of writing the elements does not change the set.

5. (a) How many words can be formed using letter of UMBRELLA using each letter at most once?

(i) If each letter must be used,

{U,M,B,R,E,L,A}
n=7
SO
NO'S OF WORD IS = 7!
7!= 7X6X5X4X3X2X1= 5040 WORD (in the umbrella letter L two time be we count once time)

(ii) If some or all the letters may be omitted

=P(7,0) + p(7,1)+P(7,2) + p(7,3)+P(7,4) + p(7,5)+P(7,6) + p(7,7)
= 1 + 7 + (7x6) + (7x6x5) + (7x6x5x4) +(7x6x5x4x3) +(7x6x5x4x3x2) + (7x6x5x4x3x2x1)
=1+7+42 +210+840 +2520 + 5040+5040
= 13700

(b) Show using truth that:

(p 📫☹ q) 📫☹ q ⇒ p V q

P	q	p -> q	(p->q)->q	p or q
TRUE	FALSE	FALSE	TRUE	TRUE
TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	TRUE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE

(c) Explain whether (p -> q) -> (q -> r) is a tautology or not.

p	q	r	p -> q	q -> r	(p -> q) - > (q -> r)
TRUE	TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE	TRUE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE

Not a Tautology 2 value is false

(d) Prove that: 1 + 2 + 3 + . . . + n = ½n(n + 1) using mathematical induction.

6. (a) How many ways are there to distribute 15 district objects

into 5 distinct boxes with:

(i) At least three empty box.

$$C(5,3) \times P(15,2) \times 2^{13} + C(5,4) \times P(15,1) \times 1^{14} + C(5,5) \times P(15,0) \times 0^{15}$$

(ii) No empty box.

$$C(15,5) \times 5^5 \times 5!$$

Ans (b)

Definition Of Multiplication Principle:-

Multiplication Principle states: If an event occurs in m ways and another event occurs independently in n ways, then the two events can occur in m × n ways.

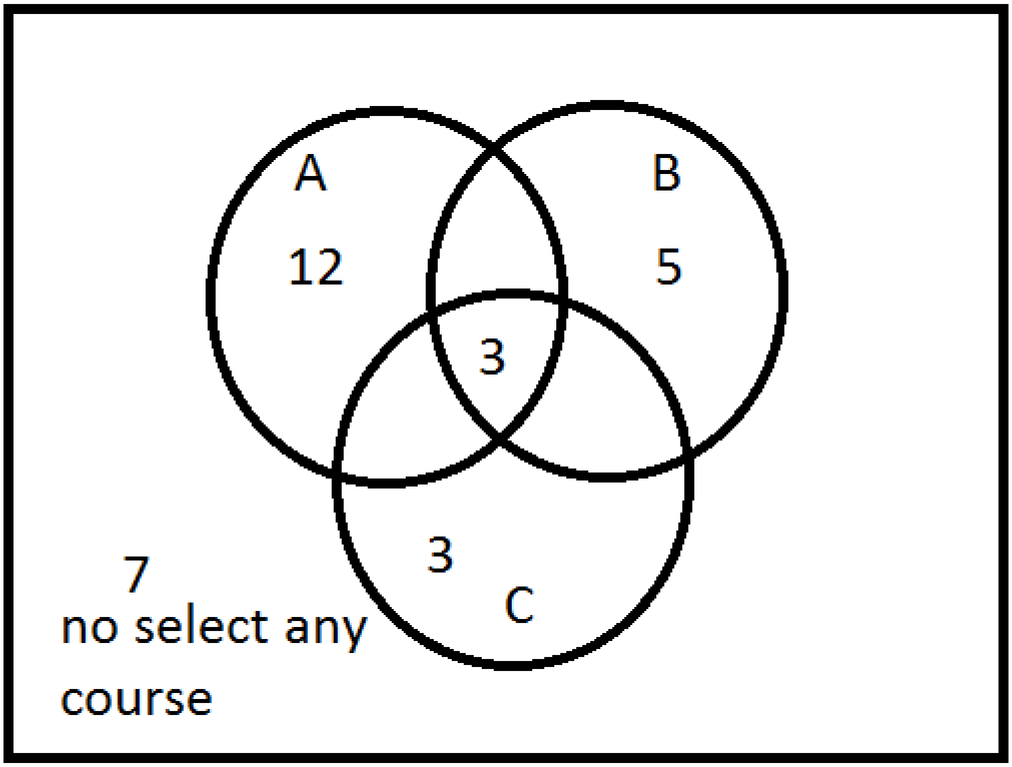
EXAMPLES OF MULTIPLICATION PRINCIPLE

A pizza corner sells pizza in 3 sizes with 3 different toppings. If Bob wants to pick one pizza with one topping, there is a possibility of 9 combinations as the total number of outcomes is equal to Number of sizes of pizza × Number of different toppings.

(c)
Set A,B and C are:
A = {1, 2, 3,5, 8, 11 12,13},
B = { 1,2, 3 ,4, 5,6 } and
C { 7,8,12, 13}.
Find A∩ B U C , A U B U C, A U B ∩ C and (B~C)

A∩ B U C ={1,2,3,5,7,8,12,13}
A U B U C={ 1,2,3,4,5,6,7,8,11,12,13}
A U B ∩ C= {8,12,13}
(B~C)= { 1,2, 3 ,4, 5,6 }

(d) Out of 30 students in college 15 takes art courses, 8 takes biology courses and 6 takes chemistry. It is also known that 3 students take all the three courses. Show that 7 or more students taken none of the course.



7. (a) Explain principle of duality with example?

Ans.

Principal of duality

According to principle of duality "Dual of one expression isobtained by replacing AND (.)withOR(+) and OR with AND togetherwith replacement of 1 with 0 and 0 with 1.

For example:

consider the expression A+B=0. The dual of this expression is obtained by replacing + with . and 0 by 1. i.e., A.B=1 is dual of A+B=0.

(b) What is power set? Write power set of set
A={1,2,3,4,5,6}.

In mathematics, the **power set** (or **powerset**) of any **set** S is the **set** of all subsets of S, including the empty **set** and S itself.

power set of A = {{123456}{12345}{23456}{34561}.....................................{ }}

2 ^6 subset

= 64 subset (dear friend make a posibal 64 set in your answer sheat)

(c) What is a function? Explain domain and range in context of function with example.

A *function* is a rule which relates the values of one variable quantity to the values of another variable quantity, and does so in such a way that the value of the second variable quantity is uniquely determined by (i.e. is a *function* of) the value of the first variable quantity.

(d) State and prove the Pigeonhole principle.

Ans.
Pigeonhole Principle

The word "pigeonhole" literally refers to the shelves in the form of square boxes or holes that were utilized to place pigeons earliar in the United States. In mathematics, there is a concept, inspired by such pigeonholes, known as pigeonhole principlewhich was introduced in 1834 by a German mathematician Peter Gustav Lejeune Dirichlet. On his name, this principle is also termed as Dirichlet principle.

Pigeonhole principle roughly states that if there are few boxes available; also, there are few objects that are greater than the total number of boxes and one needs to place objects in the given boxes, then at least one box must contain more than one such objects.

In this page below, we shall go ahead and learn about pigeonhole principle and its applications.

The definition of pigeonhole principle is that:
If "n" number of pigeons or objects are to placed in " k" number of pigeonholes or boxes; where k<n, then there must be at least one pigeonhole or box which has more than one object.

We can also define pigeonhole principle as:
If n items are to be put in k boxes, then there must be an empty hole if and only if there exists at least one hole containing more than one pigeon.

Pigeonhole principle gives rise to many useful, but simple and quite evident extensions. According to the more formal definition of an extension of this principle.

Proof

Proof of Generalized Pigeonhole Principle
In order to prove generalized pigeonhole principle, we shall use the method of induction. According to which we will assume the contradiction and prove it wrong.
Let us suppose that total "*n*" number of pigeons are to be put in "*]m*" number of pigeonholes and *n*>*m*.

Let us assume that there is no pigeonhole with at least pigeons.

In this case, each and every pigeonhole will have less than pigeons.

Therefore, we have
Number of pigeons in each pigeonhole < nm

Total number of pigeons < number of pigeonhole
Total number of pigeons < × nm

Total number of pigeons < *n*
But given that number of pigeons are strictly equal to n.
Which is a contradiction to our assumption.
Hence there exists at least one pigeonhole having at least pigeons.

8 (a) Find inverse of the following functions.

Ans.

8. (a) Find inverse of the following functions

$$y = f(x) = \frac{x^2 + 2}{x - 3}$$

$$putting\ x = y\ in\ te\ eq.$$

$$x = \frac{y^2 + 2}{y - 3}$$

$$x(y - 3) = y^2 + 2$$

$$xy - 3x = y^2 + 2$$

$$xy - y^2 = 3x + 2$$

$$\frac{y^2(x - 1)}{y} = 3x + 2$$

$$y = \frac{3x + 2}{x - 1}$$

(b) Explain circular permutation with the help of an example.

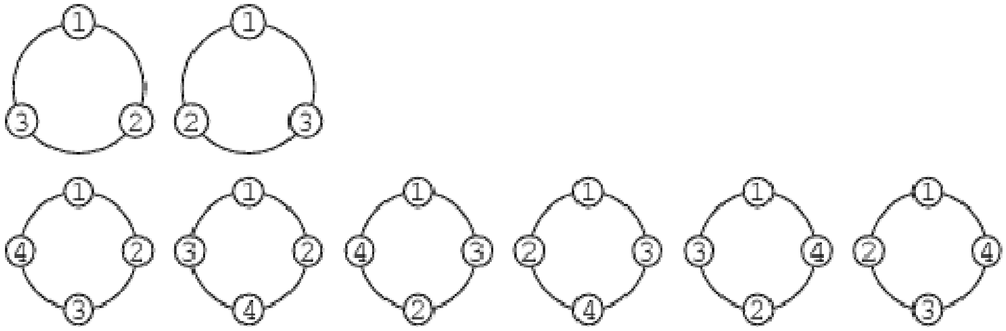
Ans.

Circular Permutation

The number of ways to arrange n distinct objects along a fixed (i.e., cannot be picked up out of the plane and turned over) circle is

$$P_n = (n - 1)!\,.$$

The number is $(n - 1)!$ instead of the usual factorial $n!$ since all cyclic permutations of objects are equivalent because the circle can be rotated.



For example, of the $3! = 6$ permutations of three objects, the $(3-1)! = 2$ distinct circular permutations are $\{1, 2, 3\}$ and $\{1, 3, 2\}$. Similarly, of the $4! = 24$ permutations of four objects, the $(4-1)! = 6$ distinct circular permutations are $\{1, 2, 3, 4\}$, $\{1, 2, 4, 3\}$, $\{1, 3, 2, 4\}$, $\{1, 3, 4, 2\}$, $\{1, 4, 2, 3\}$, and $\{1, 4, 3, 2\}$. Of these, there are only three free permutations (i.e., inequivalent when flipping the circle is allowed): $\{1, 2, 3, 4\}$, $\{1, 2, 4, 3\}$, and $\{1, 3, 2, 4\}$. The number of free circular permutations of order n is $P'_n = 1$ for $n = 1$, 2, and

$$P'_n = \frac{1}{2}(n-1)!$$

for $n \geq 3$, giving the sequence 1, 1, 1, 3, 12, 60, 360, 2520, ... (OEIS A001710).

(c) What is indirect proof? Explain with an example.

Ans.

Definition Of Indirect Proof

Indirect proof is a type of proof in which a statement to be proved is assumed false and if the assumption leads to an impossibility, then the statement assumed false has been proved to be true.

EXAMPLE OF INDIRECT PROOF

Sum of 2n even numbers is even, where n > 0. Prove the statement using an indirect proof. The first step of an indirect proof is to assume that 'Sum of even integers is odd.' That is, $2 + 4 + 6 + 8 + ... + 2n$ = an odd number $\Rightarrow 2(1 + 2 + 3 + 4 + . . . + n)$ = an odd number $\Rightarrow 2 \times \left[\frac{n(n+1)}{2}\right]$ = an odd number $\Rightarrow n(n + 1)$ = an odd number, a contradiction, because $n(n + 1)$ is always an even number. Thus, the statement is proved using an indirect proof.

(d) What is Boolean algebra?

Ans.

Boolean algebra

In mathematics and mathematical logic, Boolean algebra is the branch of algebra in which the values of the variables are the truth values true and false, usually denoted 1 and 0 respectively. Instead of elementary algebra where the values of the variables are numbers, and the main operations are addition and multiplication, the main operations of Boolean algebra are the conjunction and denoted as ∧, the disjunction or denoted as ∨, and the negation not denoted as . It is thus a formalism for describing logical relations in the same way that ordinary algebra describes numeric relations.

Boolean algebra was introduced by George Boole in his first book The Mathematical Analysis of Logic (1847), and set forth more fully in his An Investigation of the Laws of Thought (1854). According to Huntington, the term "Boolean algebra" was first suggested by Sheffer in 1913.

Boolean algebra has been fundamental in the development of digital electronics, and is provided for in all modern programming languages. It is also used in set theory and statistics.